

Deconfinement and Chiral Symmetry Restoration

Ágnes Mócsy^{†*}, Francesco Sannino[‡] and Kimmo Tuominen^{††}

[†] Institut für Theoretische Physik, J.W. Goethe-Universität, Postfach 11 19 32,
60054 Frankfurt am Main, Germany

[‡] The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

^{††} Department of Physics, P.O. Box 35, FIN-40014 University of Jyväskylä, Finland,
and Helsinki Institute of Physics, P.O. Box 64, FIN-00014 University of Helsinki,
Finland

Abstract. We illustrate why color deconfines when chiral symmetry is restored in gauge theories with quarks in the fundamental representation, and while these transitions do not need to coincide when quarks are in the adjoint representation, entanglement between them is still present.

Introduction. One of the long-standing puzzles in theoretical physics is the relation between confinement and chiral symmetry breaking.

In pure Yang-Mills theory the Z_N center of the gauge $SU(N)$ group is a global symmetry. The Polyakov loop is a gauge invariant operator charged under Z_N , whose expectation value vanishes at low temperatures, and is nonzero above a critical temperature T_χ , when the center symmetry is spontaneously broken. This feature, together with its relation to the infinitely heavy quark free energy makes the Polyakov loop suitable to be the order parameter for the deconfinement transition in pure gauge theory. We denote with χ the Polyakov loop field which is usually denoted with ℓ .

When quarks with finite masses are added to the theory in the fundamental representation of the gauge group the Z_N symmetry is not exact. QCD with massless quarks exhibits chiral symmetry. The order parameter, the chiral condensate, is zero above T_σ , where chiral symmetry is restored. Here σ is the interpolating field associated to the scalar component of the $\bar{q}q$ operator. For any finite quark mass chiral symmetry is explicitly broken. When quarks are in the adjoint representation the center group symmetry is intact. For realistic quark masses there are no exact symmetries, but one can still follow the behavior of the condensates. Analysis done on the lattice showed that with quarks in the fundamental representation deconfinement (a rise in the Polyakov loop), happens at the temperature where chiral symmetry is restored (chiral condensate decreases) $T_\chi = T_\sigma$ [1]. Lattice also revealed that when quarks are in the adjoint representation deconfinement and the chiral symmetry restoration do not happen at the same temperature, $T_\sigma \simeq 8T_\chi$ [2]. Despite the attempts to explain these behaviors [3],

* Speaker at the conference.

the underlying reasons are still unknown. Lattice simulations for two color QCD at non-zero baryon chemical potential observe deconfinement for 2 color QCD and 8 continuum flavors. Here also, the Polyakov loop rises when the chiral condensate vanishes, at the same value of the chemical potential [4].

Our goal is to provide a simple unified way to describe all of these features. We study the two color theory with N_f flavors in the chiral limit, since with only minor modifications of the effective Lagrangian we can discuss the theory with quarks in the fundamental and adjoint representation at nonzero temperature or quark chemical potential. The results presented here are based on our recent work [5] concerning the transfer of critical properties from true order parameters to non-critical fields, approach presented in [6, 7], envisioned first in [8]. For a complete review see [9]. The transfer of information is possible due to the presence of a trilinear interaction between the light order parameter and the heavy non-order parameter field, singlet under the symmetries of the order parameter field. Due to this interaction, the expectation value of the order parameter field in the symmetry broken phase induces a variation in the expectation value for the singlet field, and spatial correlators for the non-critical fields are infrared dominated.

Fundamental Representation. In two color QCD with two massless quark flavors in the fundamental representation the global symmetry group is $SU(2N_f)$ which breaks to $Sp(2N_f)$. The chiral degrees of freedom are $2N_f^2 - N_f - 1$ Goldstone fields π^a , and a scalar field σ , which is the order parameter. For $N_f = 2$ the potential is [10]:

$$V_{\text{ch}}[\sigma, \pi^a] = \frac{m^2}{2} \text{Tr} [M^\dagger M] + \lambda_1 \text{Tr} [M^\dagger M]^2 + \frac{\lambda_2}{4} \text{Tr} [M^\dagger M M^\dagger M] \quad (1)$$

with $2M = \sigma + i2\sqrt{2}\pi^a X^a$, $a = 1, \dots, 5$ and the generators $X^a \in \mathcal{A}(SU(4)) - \mathcal{A}(Sp(4))$ are provided explicitly in equation (A.5) and (A.6) of [10]. The Polyakov loop, denoted by χ , is treated as a heavy field singlet under the chiral symmetry. Its contribution to the potential in the absence of the Z_2 symmetry is

$$V_\chi[\chi] = g_0\chi + \frac{m_\chi^2}{2}\chi^2 + \frac{g_3}{3}\chi^3 + \frac{g_4}{4}\chi^4. \quad (2)$$

The interaction terms allowed by chiral symmetry are

$$V_{\text{int}}[\chi, \sigma, \pi^a] = (g_1\chi + g_2\chi^2) \text{Tr} [M^\dagger M] = (g_1\chi + g_2\chi^2) (\sigma^2 + \pi^a\pi^a). \quad (3)$$

The g_1 term plays a fundamental role. In the symmetry broken phase with $T < T_{c\sigma}$ the σ acquires a non-zero expectation value, which in turn induces a modification also for $\langle\chi\rangle$. The extremum of the linearized potential, near the phase transition where σ is small, is at

$$\langle\sigma\rangle^2 \simeq -\frac{m_\sigma^2}{\lambda}, \quad m_\sigma^2 \simeq m^2 + 2g_1\langle\chi\rangle, \quad \text{and} \quad \langle\chi\rangle \simeq -\frac{g_0}{m_\chi^2} - \frac{g_1}{m_\chi^2}\langle\sigma\rangle^2, \quad (4)$$

with $\lambda = \lambda_1 + \lambda_2$. Near T_c the mass of the order parameter field is assumed to possess the generic behavior $m_\sigma^2 \sim (T - T_c)^\nu$. For $g_1 > 0$ and $g_0 < 0$ the expectation value of χ behaves oppositely to that of σ : As the chiral condensate starts to decrease

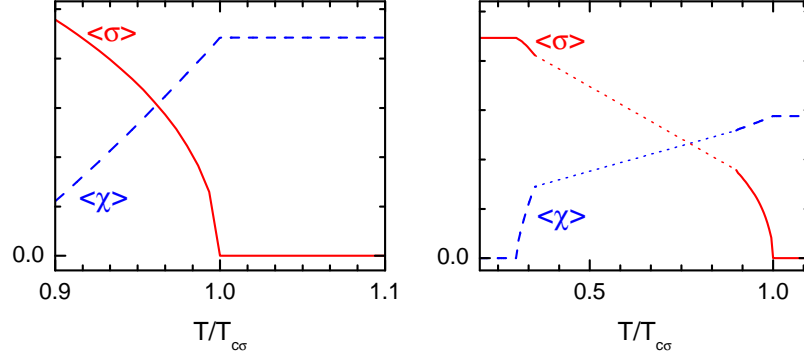


Figure 1. Left panel: Expectation values of the Polyakov loop and chiral condensate versus temperature, with massless quarks in the fundamental representation. Right panel: Same as in left panel, for the adjoint representation and $T_{c\chi} \ll T_{c\sigma}$.

towards chiral symmetry restoration, the expectation value of the Polyakov loop starts to increase, signaling the onset of deconfinement. This is illustrated in the left panel of figure 1. When applying the analysis presented in [6, 7], we find a drop near the transitions, on both sides, in the spatial two-point correlator of the Polyakov loop.

Adjoint Representation. In two color QCD with two massless Dirac quark flavors in the adjoint representation the global symmetry is $SU(2N_f)$ which breaks via a bilinear quark condensate to $O(2N_f)$. There are $2N_f^2 + N_f - 1$ Goldstone bosons, and two exact order parameter fields, the chiral σ field and the Polyakov loop χ . For $N_f = 2$ the chiral part of the potential is given by (1) with $2M = \sigma + i2\sqrt{2}\pi^a X^a$, $a = 1, \dots, 9$ and the generators $X^a \in \mathcal{A}(SU(4)) - \mathcal{A}(O(4))$ are provided explicitly in equation (A.3) and (A.5) of [10]. The Z_2 symmetric potential for the Polyakov loop is

$$V_\chi[\chi] = \frac{m_{0\chi}^2}{2}\chi^2 + \frac{g_4}{4}\chi^4, \quad (5)$$

and the only interaction term allowed by symmetries is

$$V_{\text{int}}[\chi, \sigma, \pi] = g_2\chi^2 \text{Tr} [M^\dagger M] = g_2\chi^2(\sigma^2 + \pi^a\pi^a). \quad (6)$$

Since the relevant interaction term $g_1\chi\sigma^2$ is now forbidden, one might expect no information transfer between the fields. Our analysis suggests though that this is not the case. Consider the physical case in which the deconfinement happens first [2], $T_{c\chi} \ll T_{c\sigma}$. For $T_{c\chi} < T < T_{c\sigma}$ both symmetries are broken, and the expectation values of the two order parameter fields are linked to each other:

$$\langle\sigma\rangle^2 = -\frac{m^2 + 2g_2\langle\chi\rangle^2}{\lambda} \equiv -\frac{m_\sigma^2}{\lambda}, \quad \langle\chi\rangle^2 = -\frac{m_{0\chi}^2 + 2g_2\langle\sigma\rangle^2}{g_4} \equiv -\frac{m_\chi^2}{g_4}. \quad (7)$$

The behavior of $m_\chi^2 \sim (T - T_{c\chi})^{\nu_\chi}$ and $m_\sigma^2 \sim (T - T_{c\sigma})^{\nu_\sigma}$ near $T_{c\chi}$ and $T_{c\sigma}$, respectively, combined with (7), yields near these two transitions the qualitative situation illustrated in the right panel of figure 1. On both sides of $T_{c\chi}$ ($T_{c\sigma}$) the relevant interaction term $g_2\langle\sigma\rangle\sigma\chi^2$ ($\langle\chi\rangle\chi\sigma^2$) emerges, leading to the infrared sensitive contribution $\propto \langle\sigma\rangle^2/m_\chi$

($\propto \langle \chi \rangle^2 / m_\sigma$) to the σ (χ) two-point function. Thus when $T_{c\chi} \ll T_{c\sigma}$ the two order parameter fields, a priori unrelated, do feel each other near the respective phase transitions. The existence of substructures near these transitions must be checked via lattice calculations.

Quark Chemical Potential. For two color QCD extending the discussion to finite chemical potential is straightforward. With quarks in the pseudoreal representation there is a phase transition from a quark-antiquark condensate to a diquark condensate [11]. We hence predict that when diquarks form for $\mu = m_\pi$, the Polyakov loop feels the presence of the phase transition exactly in the same manner as it feels when considering the temperature driven phase transition. This was seen in lattice simulations [4].

Discussion. Within an effective Lagrangian approach we have shown how deconfinement (a rise in the Polyakov loop) is a consequence of chiral symmetry restoration in the presence of massless quarks in the fundamental representation. We expect this to hold for small quark masses. If quark masses were very large then chiral symmetry would be badly broken, and could not be used to characterize the phase transition. But then Z_N symmetry becomes more exact, and its breaking would drive the (approximate) restoration of chiral symmetry. In the non-perturbative regime the amount of Z_N breaking is unknown, so we cannot establish which symmetry is more broken for a given quark mass. We can make a rough estimate by comparing two ratios: quark mass/confining scale, for chiral symmetry breaking, and, unless some dynamical suppression, N_f/N , for Z_N breaking [9]. A two phase transitions situation is still possible in QCD with massless quarks in the limit of large number of colors for fixed number of flavors, $N_f/N \ll 1$, but this is unnatural in the case $N_f \sim N$. Which of the underlying symmetries demands and which amends can be determined directly from the critical behavior of the spatial correlators of hadrons or of the Polyakov loop [6, 7], and by extending our analysis with the systematic study of the effects of quark masses, quark flavors, anomalies, etc. This would help to further understand the nature of phase transitions in QCD.

- [1] F. Karsch, Lect. Notes Phys. **583**, 209 (2002) [arXiv:hep-lat/0106019].
- [2] F. Karsch and M. Lutgemeier, Nucl. Phys. B **550**, 449 (1999) [arXiv:hep-lat/9812023].
- [3] An incomplete list: A. M. Polyakov, Phys. Lett. B **72**, 477 (1978); A. Casher, Phys. Lett. B **83**, 395 (1979); C. Adami, T. Hatsuda and I. Zahed, Phys. Rev. D **43**, 921 (1991); G. E. Brown *et al*, Nucl. Phys. A **560**, 1035 (1993); G. E. Brown *et al*, arXiv:hep-ph/0308147; S. Digal, E. Laermann and H. Satz, Nucl. Phys. A **702**, 159 (2002); K. Fukushima, in these proceedings.
- [4] B. Alles, M. D'Elia, M. P. Lombardo and M. Pepe, arXiv:hep-lat/0210039.
- [5] A. Mocsy, F. Sannino and K. Tuominen, accepted in PRL, arXiv:hep-ph/0308135.
- [6] A. Mocsy, F. Sannino and K. Tuominen, Phys. Rev. Lett. **91**, 092004 (2003) [arXiv:hep-ph/0301229].
- [7] A. Mocsy, F. Sannino and K. Tuominen, arXiv:hep-ph/0306069.
- [8] F. Sannino, Phys. Rev. D **66**, 034013 (2002) [arXiv:hep-ph/0204174].
- [9] A. Mocsy, F. Sannino and K. Tuominen, arXiv:hep-ph/0401149.
- [10] T. Appelquist, P. S. Rodrigues da Silva and F. Sannino, Phys. Rev. D **60**, 116007 (1999).
- [11] For a review on 2 color QCD see S. Hands, Nucl. Phys. Proc. Suppl. **106**, 142 (2002).